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# DP IB Maths: AI HL



# 4.4 Probability

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- \* 4.4.3 Sample Space Diagrams

## 4.4.1 Probability & Types of Events

# Your notes

### **Probability Basics**

#### What key words and terminology are used with probability?

- An **experiment** is a repeatable activity that has a result that can be observed or recorded
  - Trials are what we call the repeats of the experiment
- An outcome is a possible result of a trial
- An event is an outcome or a collection of outcomes
  - Events are usually denoted with capital letters: A, B, etc
  - n(A) is the number of outcomes that are included in event A
  - An event can have one or more than one outcome
- A **sample space** is the set of all possible outcomes of an experiment
  - This is denoted by U
  - n(U) is the total number of outcomes
  - It can be represented as a list or a table

#### How do I calculate basic probabilities?

- If all outcomes are **equally likely** then probability for each outcome is the same
  - Probability for each outcome is  $\frac{1}{n(U)}$
- Theoretical probability of an event can be calculated without using an experiment by dividing the number of outcomes of that event by the total number of outcomes

$$P(A) = \frac{n(A)}{n(U)}$$

- This is given in the formula booklet
- Identifying all possible outcomes either as a list or a table can help
- **Experimental probability** (also known as **relative frequency**) of an outcome can be calculated using results from an experiment by dividing its frequency by the number of trials

#### Frequency of that outcome from the trials

Relative frequency of an outcome is

Total number of trials (n)

#### How do I calculate the expected number of occurrences of an outcome?

- Theoretical probability can be used to calculate the expected number of occurrences of an outcome from n trials
- If the probability of an outcome is p and there are n trials then:
  - The expected number of occurrences is **np**
  - This does not mean that there will exactly np occurrences

• If the experiment is repeated multiple times then we expect the number of occurrences to average out to be np

# Your notes

#### What is the complement of an event?

- The probabilities of all the outcomes add up to 1
- Complementary events are when there are two events and exactly one of them will occur
  - One event has to occur but both events can not occur at the same time
- The complement of event A is the event where event A does not happen
  - This can be thought of as **not A**
  - This is denoted A'

$$P(A) + P(A') = 1$$

- This is in the formula booklet
- It is commonly written as P(A') = 1 P(A)

#### What are different types of combined events?

- The intersection of two events (A and B) is the event where both A and B occur
  - This can be thought of as **A** and **B**
  - This is denoted as  $A \cap B$
- The union of two events (A and B) is the event where A or B or both occur
  - This can be thought of as **A or B**
  - $\blacksquare$  This is denoted  $A \cup B$
- The event where A occurs given that event B has occurred is called **conditional probability** 
  - This can be thought as **A given B**
  - This is denoted  $A \mid B$

#### How do I find the probability of combined events?

• The probability of A or B (or both) occurring can be found using the formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- This is given in the **formula booklet**
- You subtract the probability of A and B both occurring because it has been included twice (once in P(A) and once in P(B))
- The probability of A and B occurring can be found using the formula

$$P(A \cap B) = P(A)P(B|A)$$

- A rearranged version is given in the formula booklet
- Basically you multiply the probability of A by the probability of B then happening



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# Examiner Tip

 In an exam drawing a Venn diagram or tree diagram can help even if the question does not ask you to



Dave has two fair spinners, A and B. Spinner A has three sides numbered 1, 4, 9 and spinner B has four sides numbered 2, 3, 5, 7. Dave spins both spinners and forms a two-digit number by using the spinner A for the first digit and spinner B for the second digit.

 ${\it T}$  is the event that the two-digit number is a multiple of 3.

List all the possible two-digit numbers.

	2	3	5	7
ı	12	13	15	17
4	42	43	45	47
9	92	93	95	97

Find P(T). b)

$$P(T) = \frac{n(T)}{n(U)} \leftarrow \frac{Number of multiples of 3}{Total number of outcomes}$$
  
{12, 15, 42, 45, 93} are the multiples of 3  
 $P(T) = \frac{5}{12}$ 

c) Find P(T').

$$P(T) + P(T') =$$
  $\Rightarrow$   $P(T') = |-P(T)$ 

$$P(T') = 1 - \frac{5}{12}$$

$$P(T') = \frac{7}{12}$$





## **Independent & Mutually Exclusive Events**

#### What are mutually exclusive events?

- Two events are mutually exclusive if they cannot both occur
  - For example: when rolling a dice the events "getting a prime number" and "getting a 6" are mutually exclusive
- If A and B are mutually exclusive events then:
  - $P(A \cap B) = 0$

#### What are independent events?

- Two events are independent if one occurring does not affect the probability of the other occurring
  - For example: when flipping a coin twice the events "getting a tails on the first flip" and "getting a tails on the second flip" are independent
- If A and B are independent events then:
  - P(A|B) = P(A) and P(B|A) = P(B)
- If A and B are independent events then:
  - $P(A \cap B) = P(A)P(B)$ 
    - This is given in the formula booklet
    - This is a useful formula to test whether two events are statistically independent

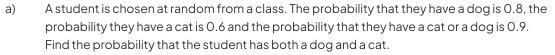
#### How do I find the probability of combined mutually exclusive events?

• If A and B are **mutually exclusive** events then

$$P(A \cup B) = P(A) + P(B)$$

- This is given in the **formula booklet**
- This occurs because  $P(A \cap B) = 0$
- For any two events A and B the events  $A \cap B$  and  $A \cap B'$  are **mutually exclusive** and A is the **union** of these two events
  - $P(A) = P(A \cap B) + P(A \cap B')$ 
    - This works for any two events A and B





b) Two events, Q and R, are such that P(Q) = 0.8 and  $P(Q \cap R) = 0.1$ . Given that Q and R are independent, find P(R).

Q and R independent 
$$\Rightarrow$$
 P(Q n R) = P(Q)P(R)  
0.1 = 0.8 × P(R)  $\therefore$  P(R) =  $\frac{0.1}{0.8}$   
P(R) = 0.125 or  $\frac{1}{8}$ 

Two events, S and T, are such that P(S) = 2P(T). Given that S and T are mutually exclusive and that  $P(S \cup T) = 0.6$  find P(S) and P(T).

S and T mutually exclusive 
$$\Rightarrow$$
 P(S U T) = P(S) + P(T)  
0.6 = P(S) + P(T)  
0.6 = 2P(T) + P(T)  
0.6 = 3P(T)  
P(T) = 0.2 and P(S) = 0.4



# 4.4.2 Conditional Probability

# Your notes

## **Conditional Probability**

#### What is conditional probability?

- Conditional probability is where the probability of an event happening can vary depending on the outcome of a prior event
- The event A happening **given that** event B has happened is denoted A B
- A common example of conditional probability involves selecting multiple objects from a bag without replacement
  - The probability of selecting a certain item changes depending on what was selected before
    - This is because the total number of items will change as they are not replaced once they have been selected

#### How do I calculate conditional probabilities?

- Some conditional probabilities can be calculated by using counting outcomes
  - Probabilities without replacement can be calculated like this
  - For example: There are 10 balls in a bag, 6 of them are red, two of them are selected without replacement
    - To find the probability that the second ball selected is red given that the first one is red count how many balls are left:
    - A red one has already been selected so there are 9 balls left and 5 are red so the probability is

 $\frac{5}{9}$ 

- You can use sample space diagrams to find the probability of A given B:
  - reduce your sample space to just include outcomes for event B
  - find the proportion that also contains outcomes for event A
- There is a formula for conditional probability that you can use

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

- This is given in the **formula booklet**
- This can be rearranged to give  $P(A \cap B) = P(B)P(A \mid B)$

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#### Worked example

In a class of 30 students: 19 students have a dog, 17 students have a cat and 11 have both a dog and a cat. One student is selected at random.

Find the probability that the student has a dog. a)

Let D be event "has a dog" and C be "has a cat"
$$P(D) = \frac{n(D)}{n(U)} \leftarrow \text{Number who have dogs}$$

$$P(D) = \frac{19}{3D}$$

b) Find the probability that the student has a dog given that they have a cat.

It have a cat of which II also have a dog
$$P(D|C) = \frac{11}{17} \quad \text{Could also use} \quad P(D|C) = \frac{P(D|C)}{P(C)}$$

c) Find the probability that the student has a cat given that they have a dog.

19 have a dog of which II also have a cat
$$P(C|D) = \frac{11}{19} \quad \text{(ould also use } P(C|D) = \frac{P(C|D)}{P(D)}$$





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# 4.4.3 Sample Space Diagrams

# Your notes

### **Venn Diagrams**

#### What is a Venn diagram?

- A Venn diagram is a way to illustrate events from an experiment and are particularly useful when there is an overlap between possible outcomes
- A Venn diagram consists of
  - a rectangle representing the sample space (U)
    - The rectangle is labelled U
    - Some mathematicians instead use S or  $\xi$
  - a circle for each event
    - Circles may or may not overlap depending on which outcomes are shared between events
- The numbers in the circles represent either the **frequency** of that event or the **probability** of that event
  - If the frequencies are used then they should add up to the total frequency
  - If the probabilities are used then they should add up to 1

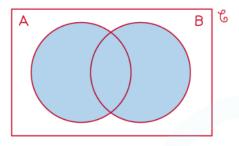
#### What do the different regions mean on a Venn diagram?

- A' is represented by the regions that are **not in** the A circle
- $A \cap B$  is represented by the region where the A and B circles **overlap**
- $A \cup B$  is represented by the regions that **are in** A or B or both
- Venn diagrams show 'AND' and 'OR' statements easily
- Venn diagrams also instantly show **mutually exclusive** events as these circles will **not overlap**
- Independent events can not be instantly seen
  - You need to use probabilities to deduce if two events are independent

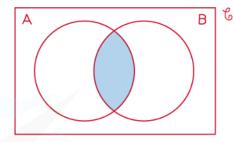


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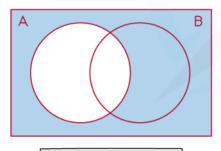






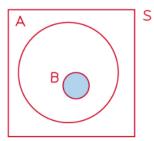


A∩B (INTERSECTION)
"A AND B"

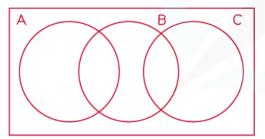


A' (COMPLEMENT)
"NOT A"

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THE BUBBLE FOR EVENT B LIES ENTIRELY IN THE BUBBLE FOR EVENT A IF EVENT B OCCURS, SO DOES EVENT A (BUT NOT NECESSARILY VICE VERSA)

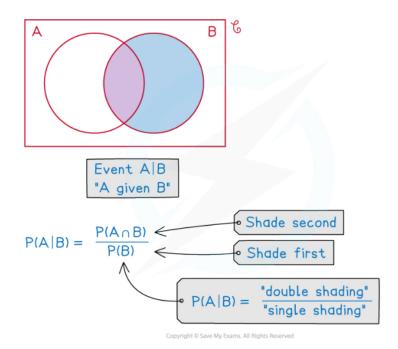


THE BUBBLES FOR EVENTS
A AND C DO NOT OVERLAP:
THEY ARE MUTUALLY EXCLUSIVE

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#### How do I solve probability problems involving Venn diagrams?

- Draw, or add to a given Venn diagram, filling in as many values as possible from the information provided in the guestion
- It is usually helpful to work from the centre outwards
  - Fill in **intersections** (overlaps) first
- If two events are independent you can use the formula
  - $P(A \cap B) = P(A)P(B)$
- To find the conditional probability  $P(A \mid B)$ 
  - Add together the frequencies/probabilities in the B circle
    - This is your denominator
  - Out of those frequencies/probabilities add together the ones that are also in the A circle
    - This is your numerator
  - Evaluate the fraction



# Examiner Tip

- If you struggle to fill in a Venn diagram in an exam:
  - Label the missing parts using algebra
  - Form equations using known facts such as:
    - the sum of the probabilities should be 1
    - $P(A \cap B) = P(A)P(B)$  if A and B are independent events

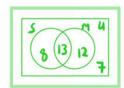


40 people are asked if they have sugar and/or milk in their coffee. 21 people have sugar, 25 people have milk and 7 people have neither.

Draw a Venn diagram to represent the information.

$$(21-x)+x+(25-x)+7=40$$

$$53 - \alpha = 40$$
 .:  $\alpha = 13$ 



b) One of the 40 people are randomly selected, find the probability that they have sugar but not milk with their coffee.

$$P(5 \cap M') = \frac{8}{40}$$

 $P(S \cap M') = \frac{8}{40}$  Remember to write as a fraction of the total

$$P(S \cap M') = \frac{1}{5}$$

Given that a person who has sugar is selected at random, find the probability that they have milk c) with their coffee.

$$P(M|S) = \frac{13}{21}$$



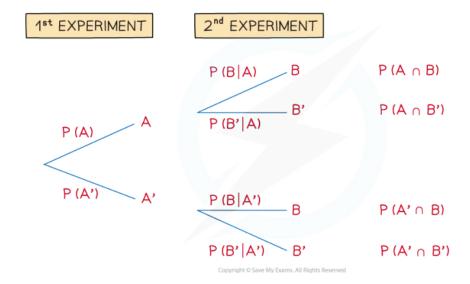
### Tree Diagrams

#### What is a tree diagram?

- A tree diagram is another way to show the outcomes of combined events
  - They are very useful for intersections of events
- The events on the branches must be **mutually exclusive** 
  - Usually they are an event and its complement
- The probabilities on the second sets of branches can depend on the outcome of the first event
  - These are conditional probabilities
- When selecting the items from a bag:
  - The second set of branches will be the **same** as the first if the items **are replaced**
  - The second set of branches will be the **different** to the first if the items **are not replaced**

#### How are probabilities calculated using a tree diagram?

- To find the probability that two events happen together you multiply the corresponding probabilities on their branches
  - It is helpful to find the probability of all combined outcomes once you have drawn the tree
- To find the probability of an event you can:
  - add together the probabilities of the combined outcomes that are part of that event
    - For example:  $P(A \cup B) = P(A \cap B) + P(A \cap B') + P(A' \cap B)$
  - **subtract** the probabilities of the combined outcomes that are not part of that event from 1
    - For example:  $P(A \cup B) = 1 P(A' \cap B')$



#### Do I have to use a tree diagram?

- If there are **multiple events** or trials then a tree diagram can get big
- You can break down the problem by using the words AND/OR/NOT to help you find probabilities without a tree





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• You can speed up the process by only drawing parts of the tree that you are interested in

### Which events do I put on the first branch?

- If the events A and B are **independent** then the **order does not matter**
- If the events A and B are **not independent** then the **order does matter** 
  - If you have the probability of **A given B** then put **B on the first set** of branches
  - If you have the probability of **B given A** then put **A on the first set** of branches

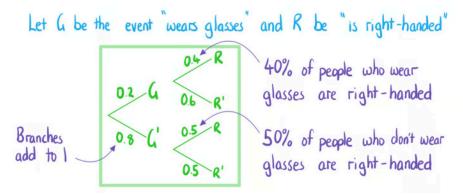
# Examiner Tip

- In an exam do not waste time drawing a full tree diagram for scenarios with lots of events unless the question asks you to
  - Only draw the parts that you are interested in

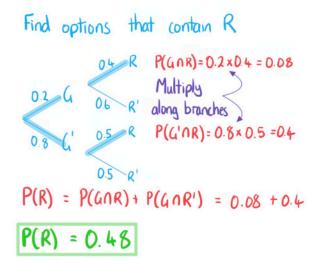


20% of people in a company wear glasses. 40% of people in the company who wear glasses are righthanded. 50% of people in the company who don't wear glasses are right-handed.

Draw a tree diagram to represent the information.



One of the people in the company are randomly selected, find the probability that they are b) right-handed.



Given that a person who is right-handed is selected at random, find the probability that they c) wear glasses.



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$$P(\alpha|R) = \frac{P(\alpha nR)}{P(R)} = \frac{0.08}{0.48}$$

$$P(\alpha|R) = \frac{1}{6}$$

$$P(G|R) = \frac{1}{6}$$

